Distributing Multipartite Entanglement over Noisy Quantum Networks

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UMass/CQN Quantum Network Science Seminar

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Intro

- Distributing Multipartite Entanglement

  - Good for: Distributed Protocols

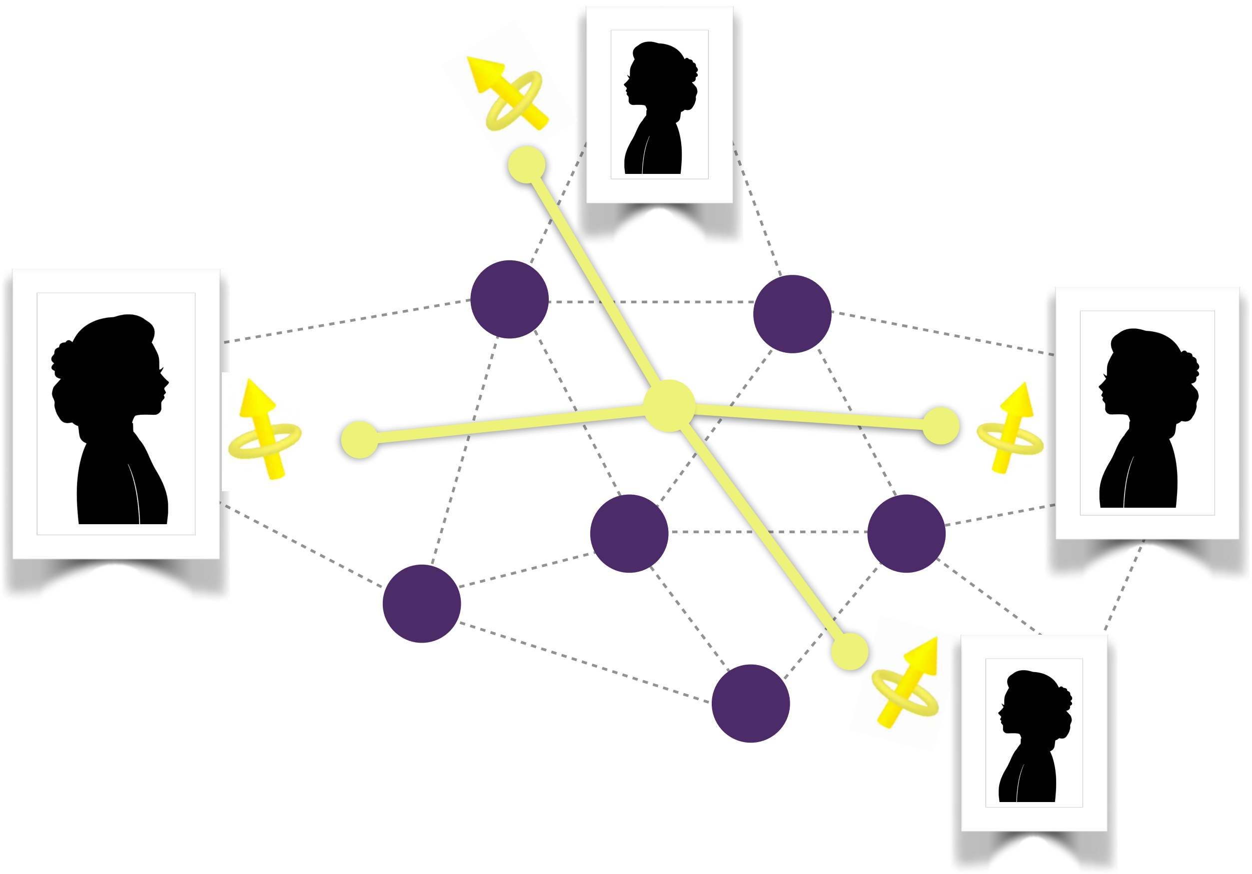
    A. Distributed Quantum Sensing

    B. Distributed Quantum Computation

    C. Multi-party Quantum Communication

IMPORTANT: Fidelity, Rate, Capacity…

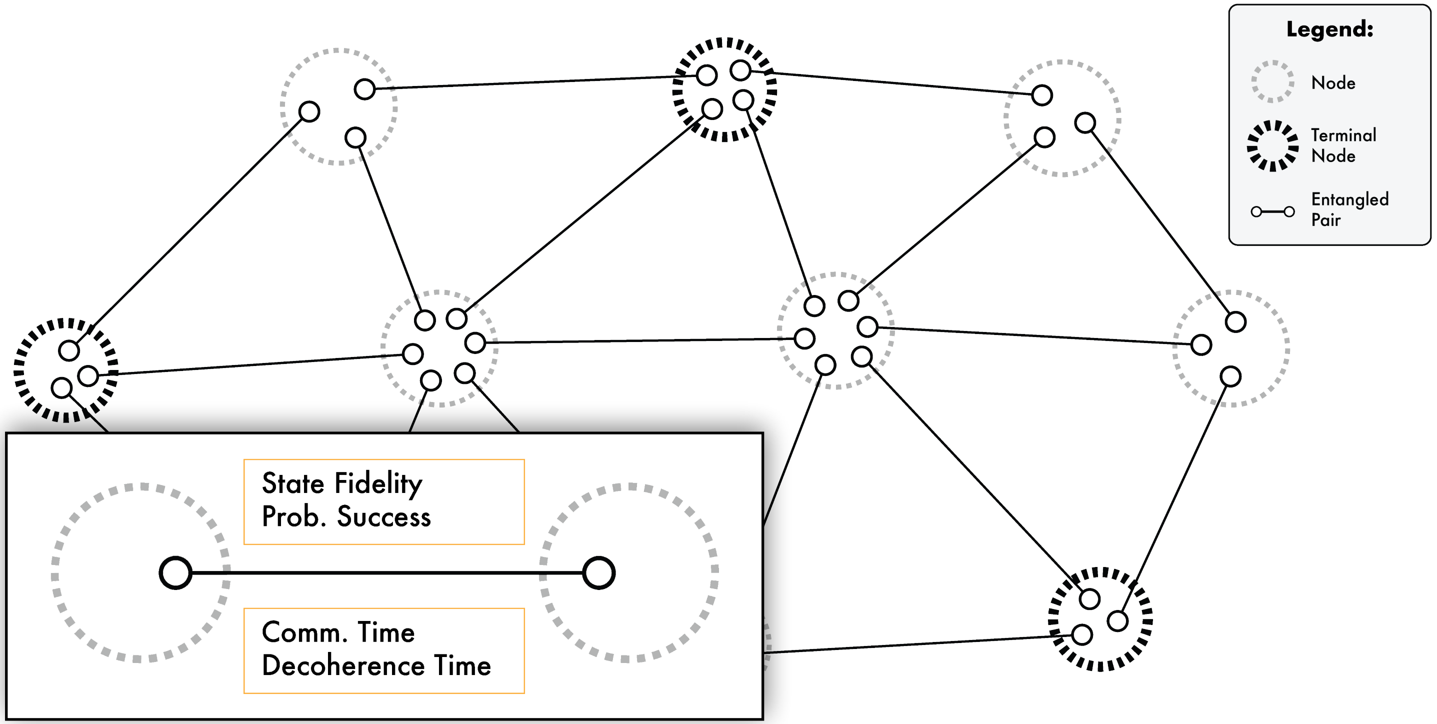
Distributing Multipartite Entanglement
over Noisy Quantum Networks



- **Multipartite Entanglement**
  - GHZ states!
  - W states
  - Graph states
  - ...

- Explore new applications not attainable using only bipartite entanglement - usually between multiple parties (>2)

Distributing Multipartite Entanglement over Noisy Quantum Networks



- **Model of Quantum Network:**
  - Nodes (terrestrial, space,...)
  - Links (entangled pairs,...)
- **Parameters:**
  - Characterizing the physical implementation
- **Protocols for distribution:**
  - Bipartite (entanglement swapping,...)
  - Multipartite (graph states,...)

Goals and Motivation

Find the optimal way to distribute multipartite entanglement over a noisy quantum network

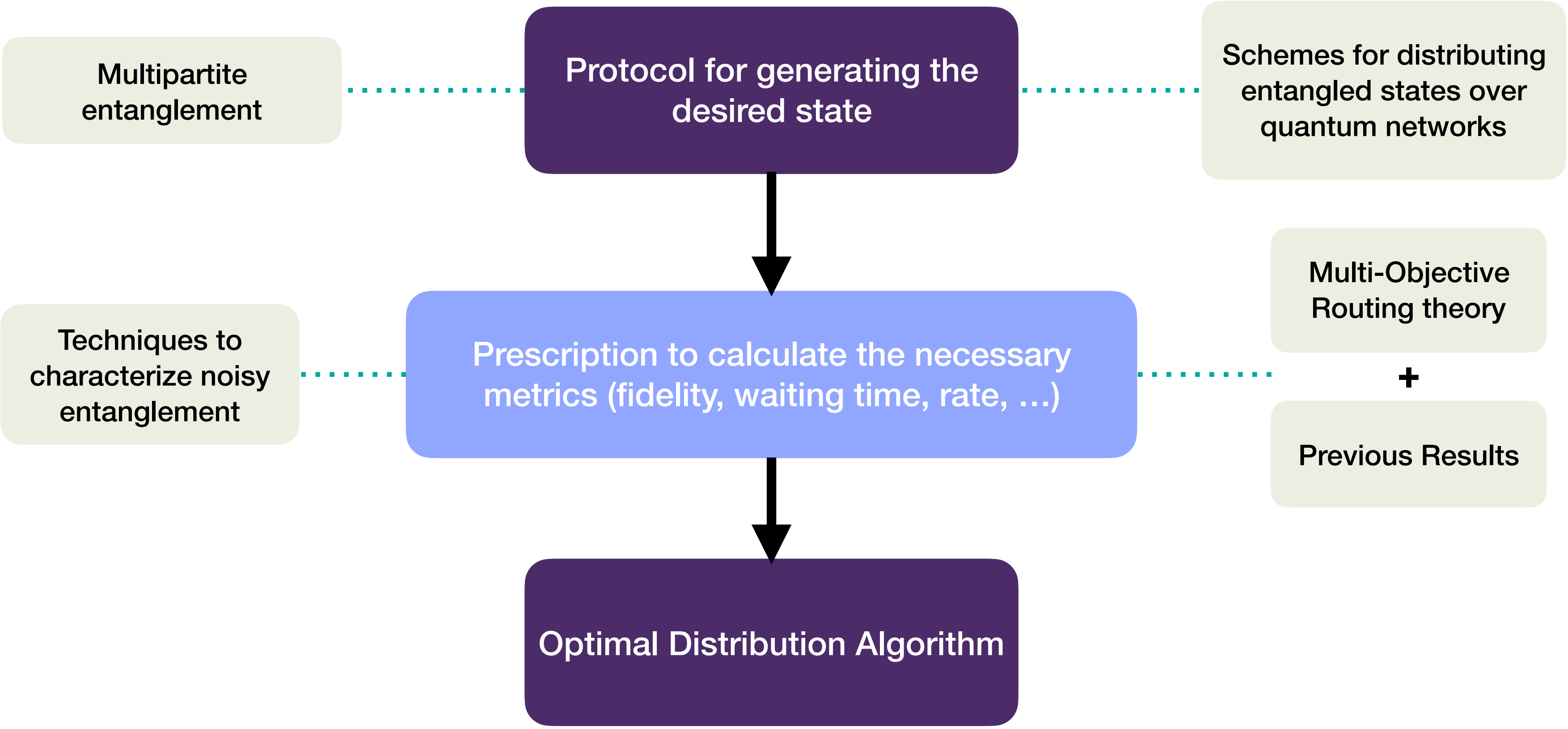
We wanted to include:

- More realistic models of quantum networks with added complications
  - Probabilistic entanglement generation and swapping protocols
  - Noisy entanglement generation and decohering memories
  - ...

And finally provide algorithms capable of solving this problem.

Overview

State of the Art



Meignant, C., Markham, D., & Grosshans, F. (2018) Phys. Rev. A
S. Brand, T. Coopmans, and D. Elkouss (2020) IEEE J. SAC

Bäuml, S., Azuma, K., Kato, G., & Elkouss, D. (2020) Communications Physics
K. Chakraborty, D. Elkouss, B. Rijsman, and S. Wehner (2020) IEEE TQE

What we used to model the network

- ❖ Description of noise channels
- ❖ Stochastic behavior of generating and swapping entanglement
- ❖ Schemes for distributing entanglement

Noise Channels

Entanglement Generation:

The depolarising channel is a quantum channel described by:

$$\mathscr{D}_i(\rho) = p\rho + \frac{1-p}{3} \cdot (\hat{X}_i\rho\hat{X}_i + \hat{Y}_i\rho\hat{Y}_i + \hat{Z}_i\rho\hat{Z}_i) \quad \equiv \quad \mathscr{D}_i(\rho) = \frac{1+2F}{3}\rho + \frac{2(1-F)}{3}\,\Lambda_i(\hat{Y}_i\rho\hat{Y}_i)$$

But one could consider additional noise channels (*e.g.* dephasing channels,...)

Quantum Memory Decoherence:

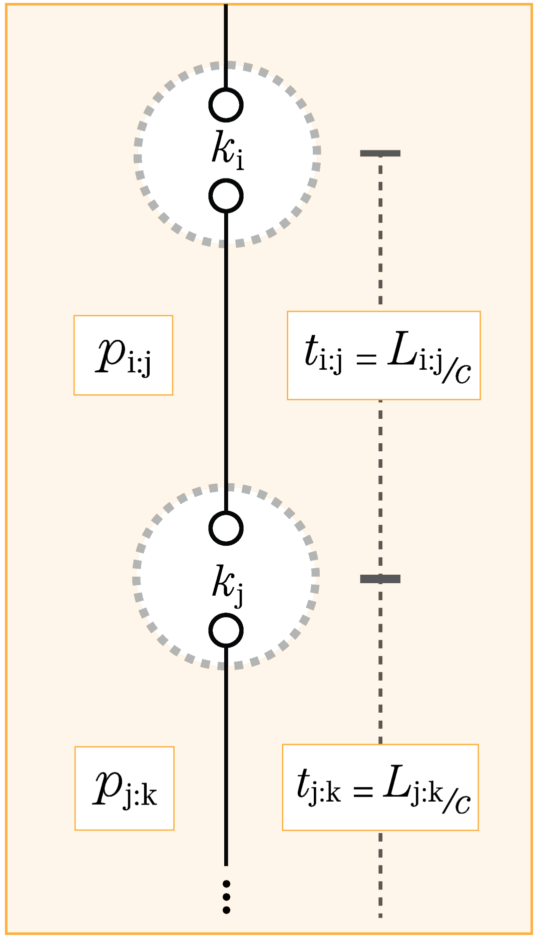
Again, a depolarizing channel, but the probability of error is parametrized as a waiting time function and naturally depends on the memory coherence time:

$$\frac{4F-1}{3} = \lambda \mapsto \lambda \cdot e^{-t_{\mathsf{wait}}/T_{\mathsf{coh}}}$$

What we used to model the network

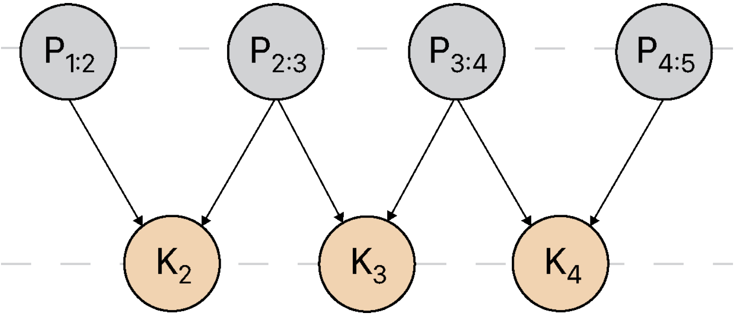
- ❖ Description of noise channels
- ❖ Stochastic behavior of generating and swapping entanglement
- ❖ Schemes for distributing entanglement

Probabilistic Processes



Both entanglement generation and entanglement swapping can be described by a geometric distribution of tries until first success.

If intermediary successes are not saved, then this two-step process is also geometrically distributed.



$$\implies p_{m:n} = \prod_{i:j \in m:n} p_{i:j} \prod_{j \in m:n \setminus m,n} k_j$$

S. Brand, T. Coopmans, and D. Elkouss (2020) IEEE J. SAC

What we used to model the network

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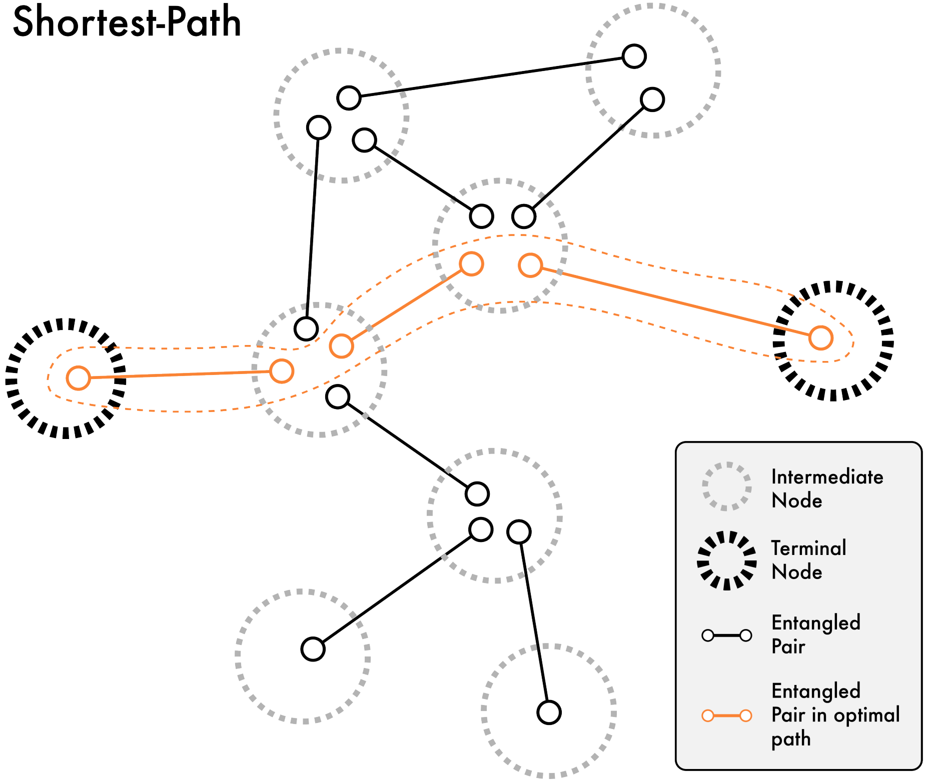
- ❖ Schemes for distributing entanglement

Schemes for Distribution

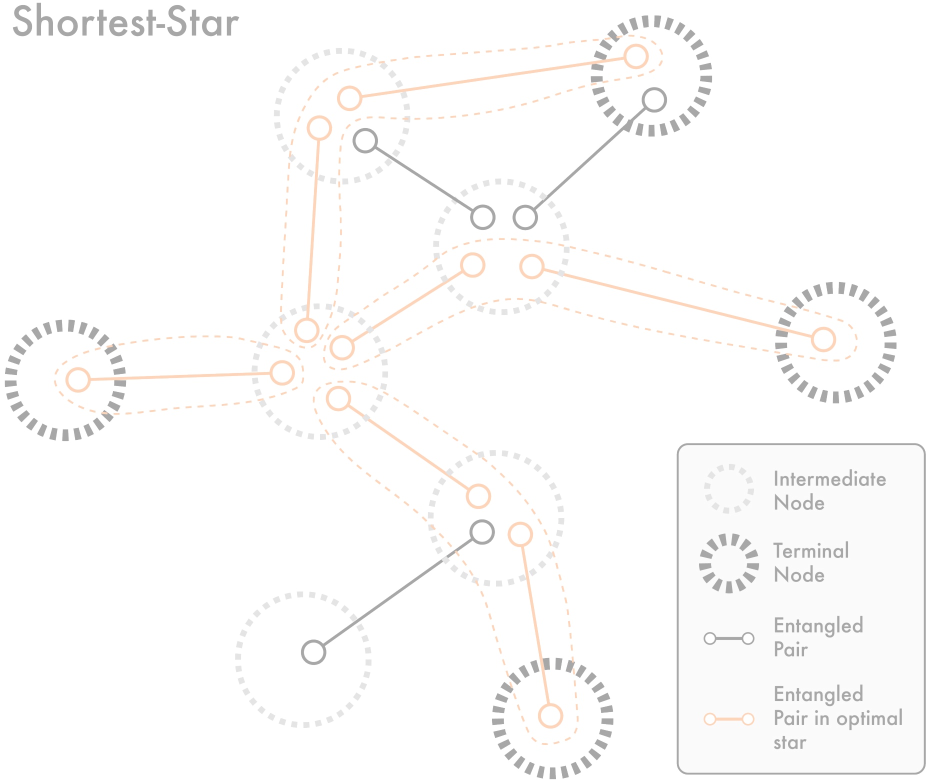
Bipartite Scenario

Shortest-Path

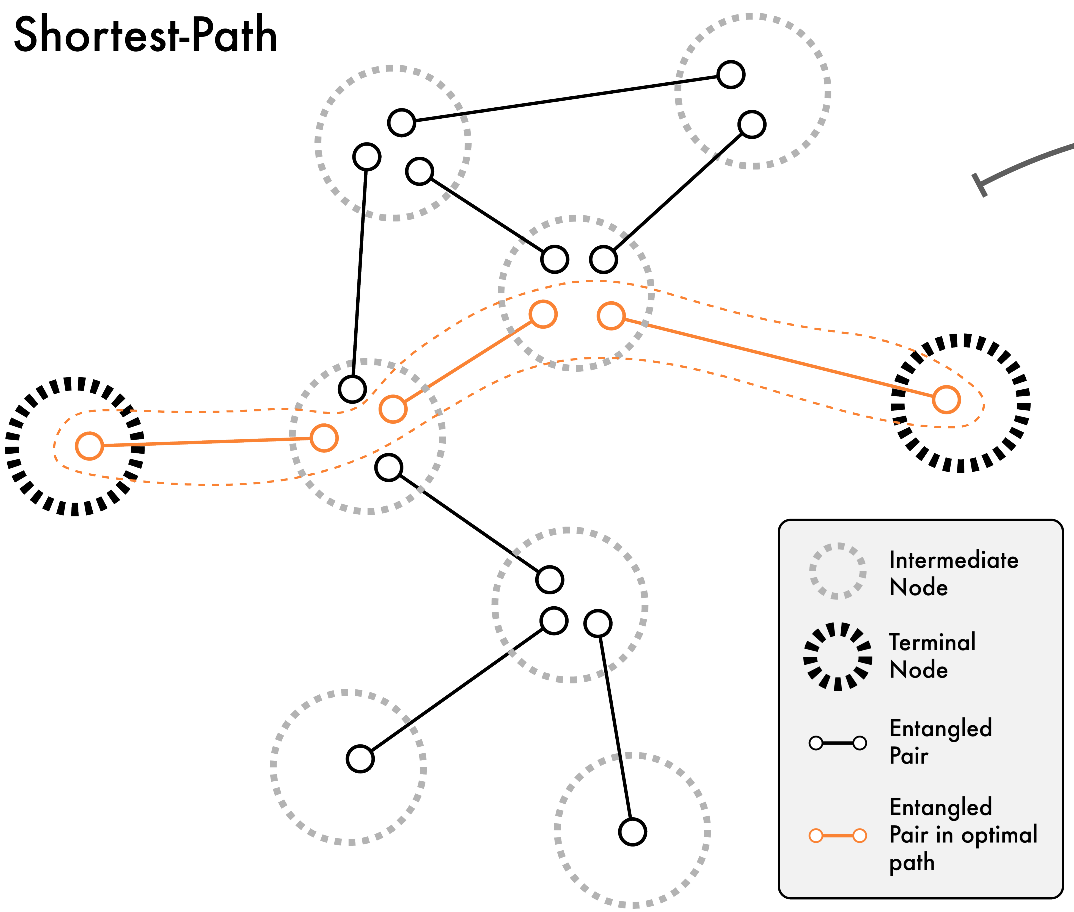


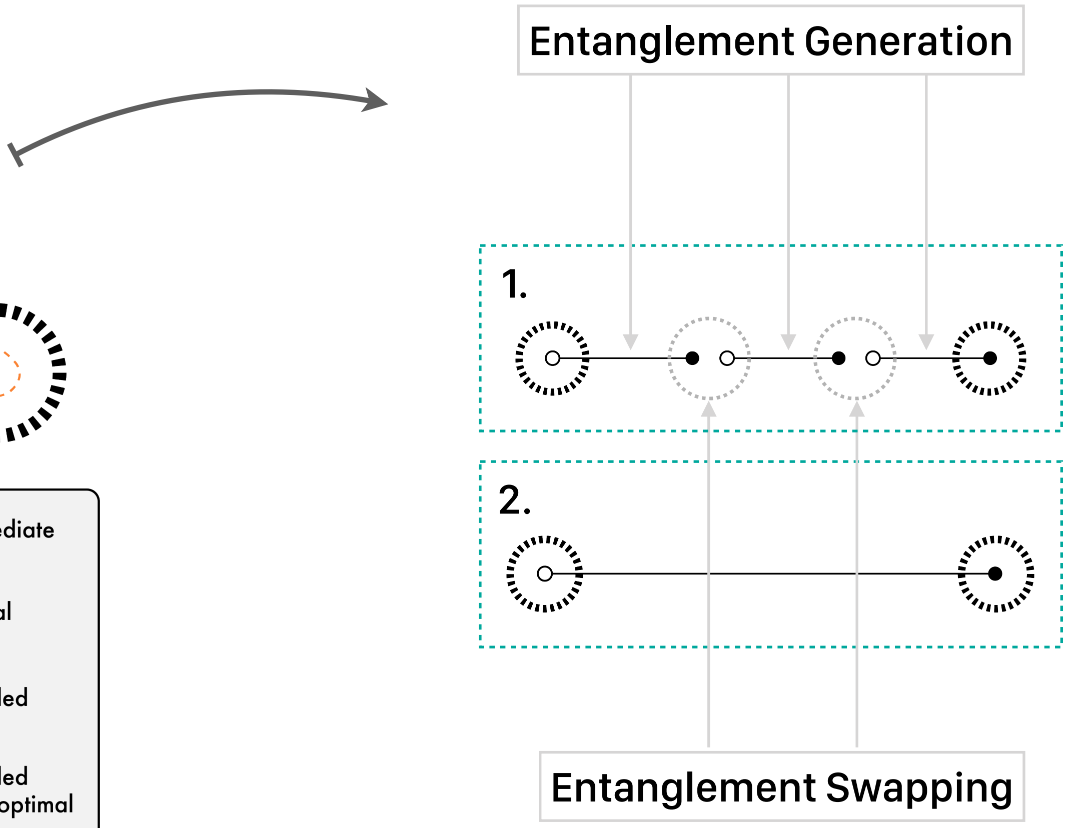
Multipartite Scenario

Shortest-Star



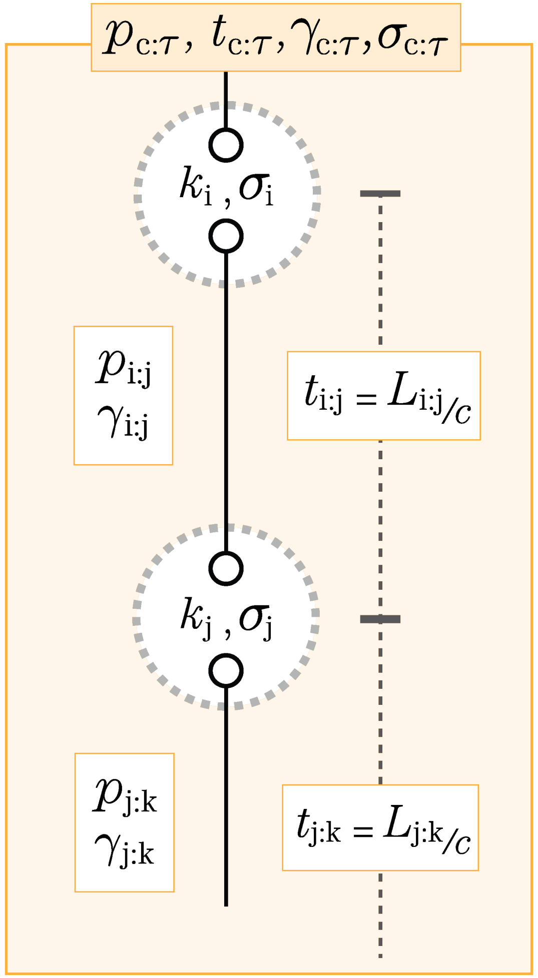
Schemes for Distribution - Swapping





No-Cloning!

Bipartite Distributions Metrics



For each path:

Probability of success

$$p_{m:n} = \prod_{i:j \in m:n} p_{i:j} \prod_{j \in m:n \setminus m,n} k_j$$

Communications Times

$$t_{m:n} = 2 \sum_{i:j \in m:n} t_{i:j}$$

Fidelity

$$\gamma_{m:n} = \begin{cases} \prod_{i:j \in m:n} \gamma_{i:j}\ , & \gamma_{m:n} \geq 1/3 \\ 0\ , & \gamma_{m:n} < 1/3. \end{cases}$$

Memory decoherence

$$\frac{1}{\sigma_{m:n}} = \sum_{i \in m:n} \frac{2}{\sigma_i}$$

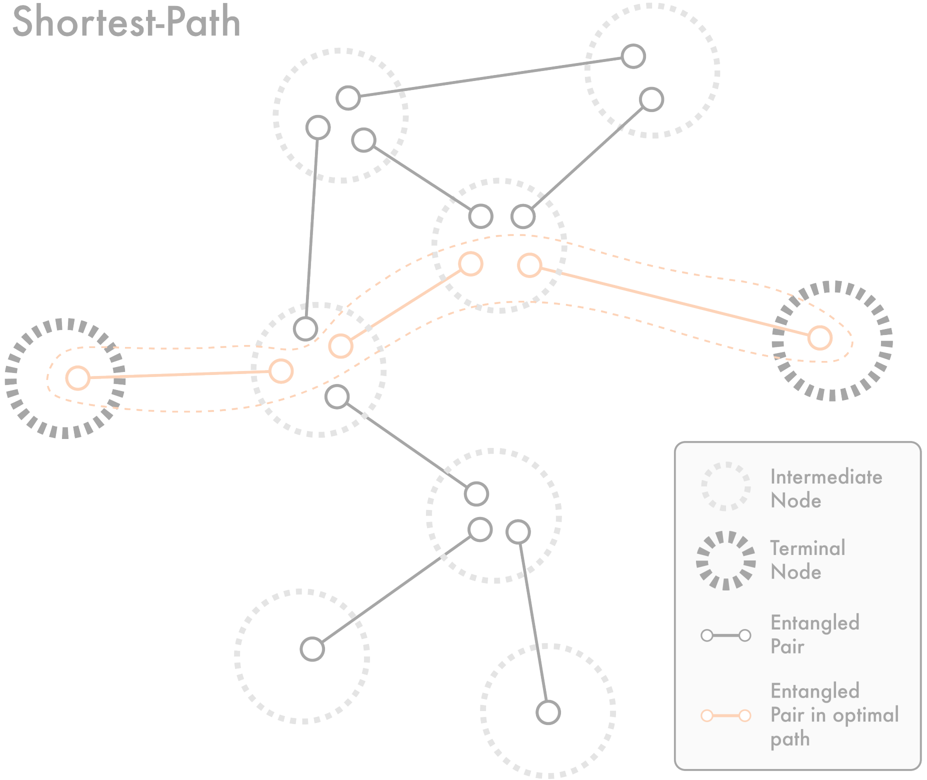
$$[p_{m:n}, t_{m:n}, \gamma_{m:n}, \sigma_{m:n}] \mapsto [p_{m:n}, t_{m:n}, F_{m:n}]$$

$$\frac{4F_{m:n} - 1}{3} = \gamma_{m:n} e^{-t_{m:n}/\sigma_{m:n}}$$

Schemes for Distribution

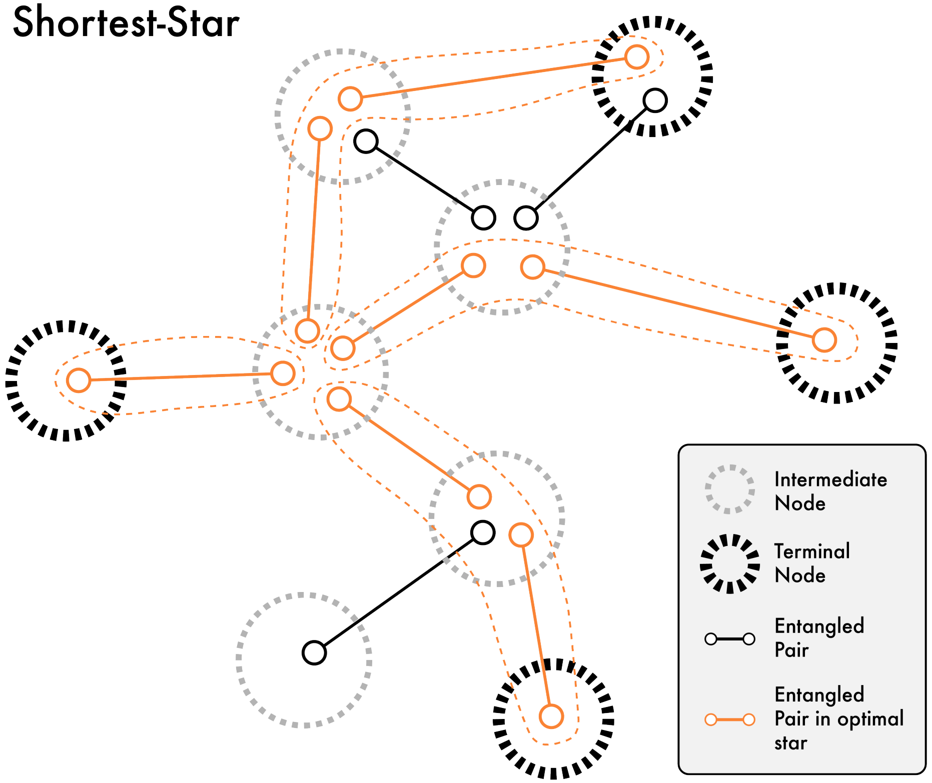
Bipartite Scenario

Shortest-Path



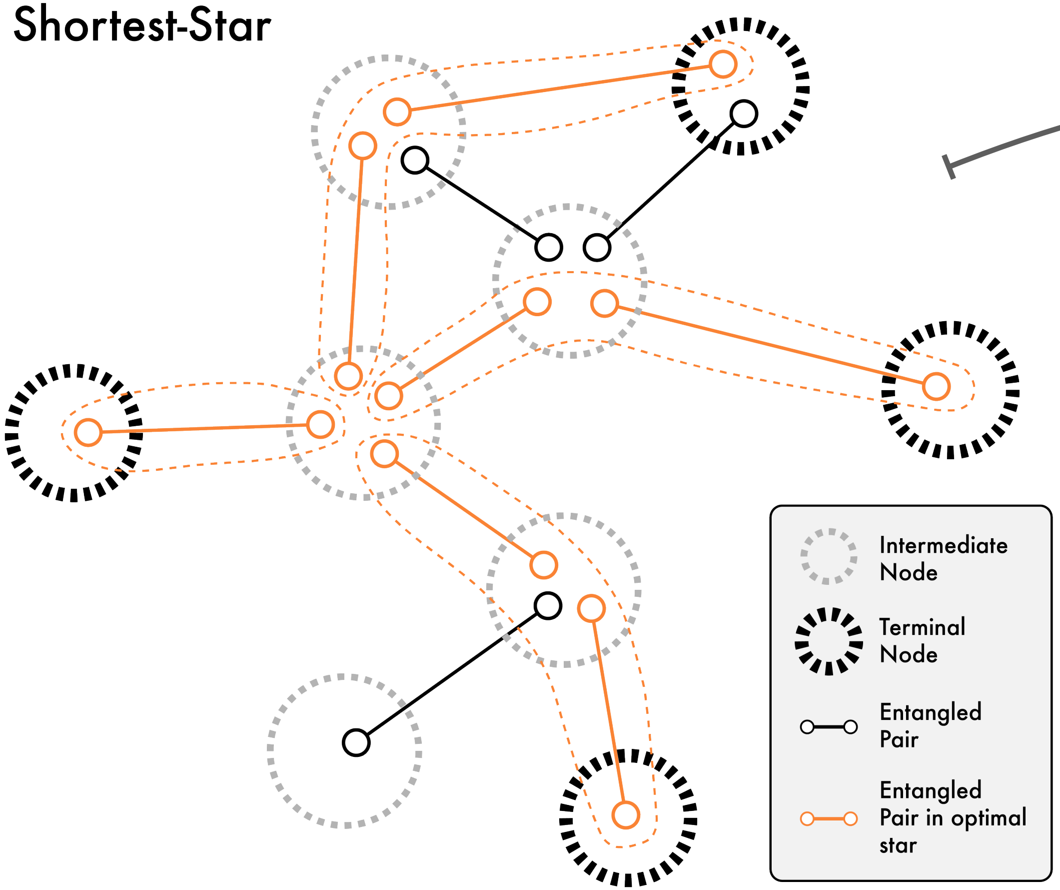
Multipartite Scenario

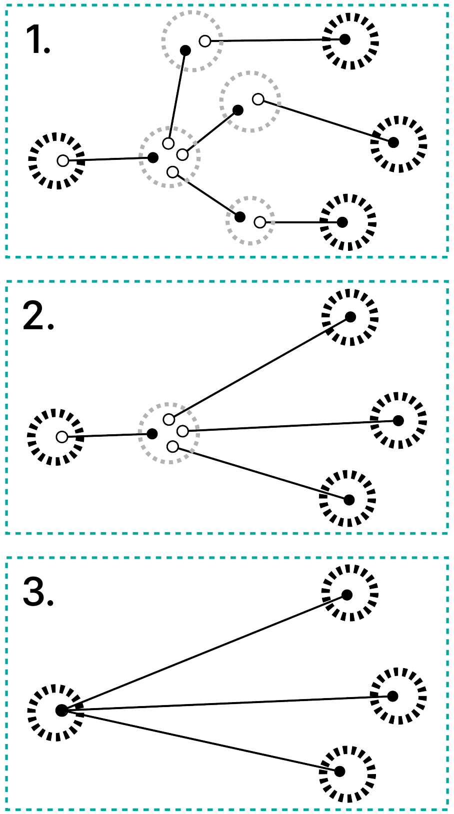
Shortest-Star



Schemes for Distribution - Star Scheme

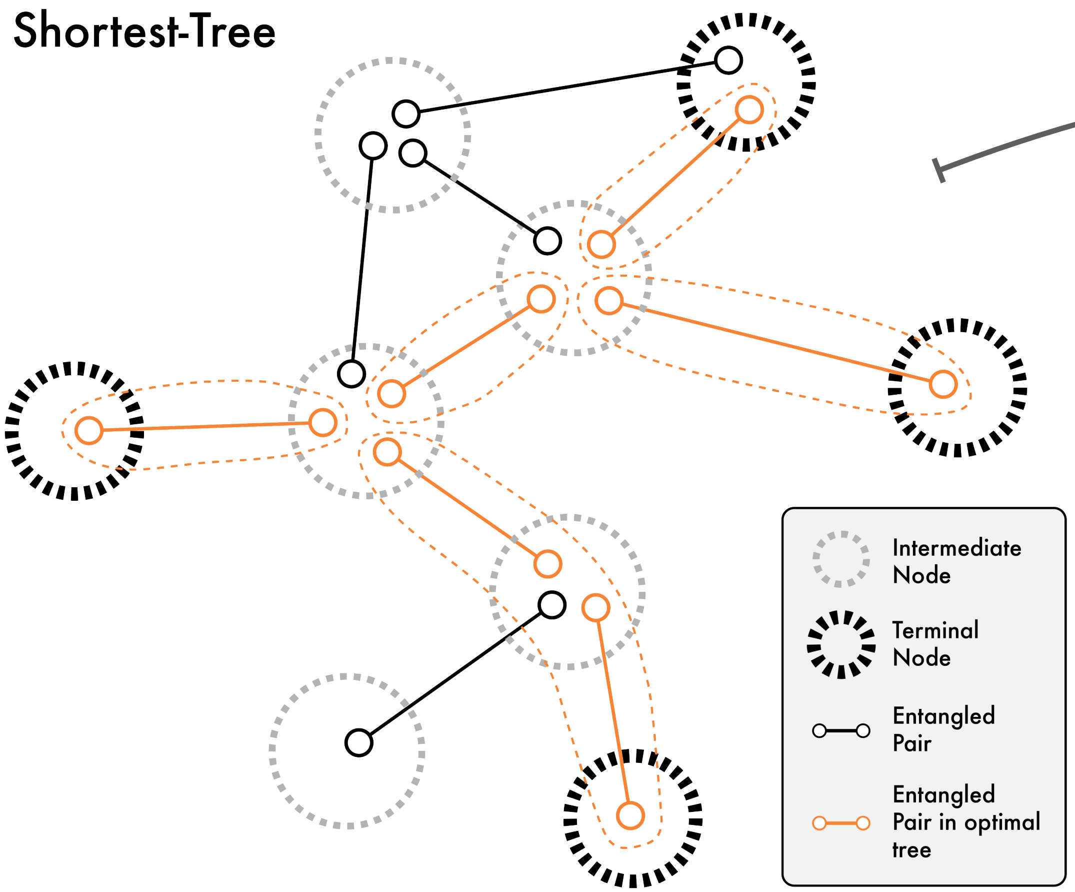
Shortest-Star

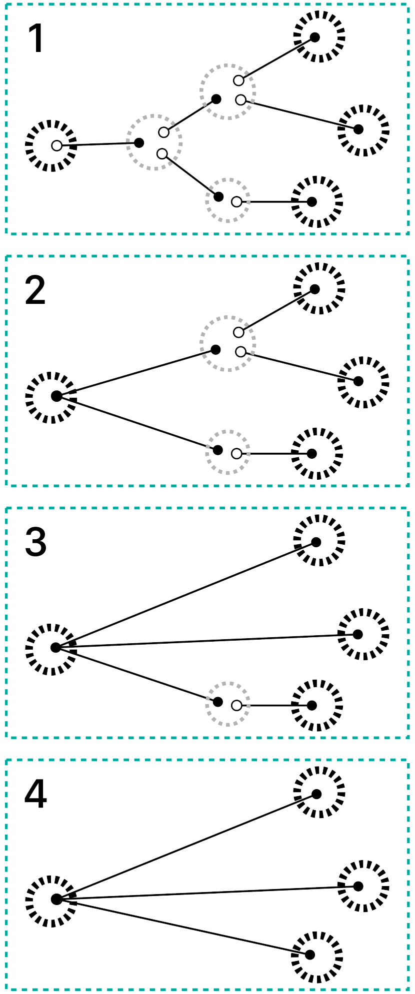




Distributes any state ≡ multiparty teleportation at center node

Schemes for Distribution - Tree Scheme

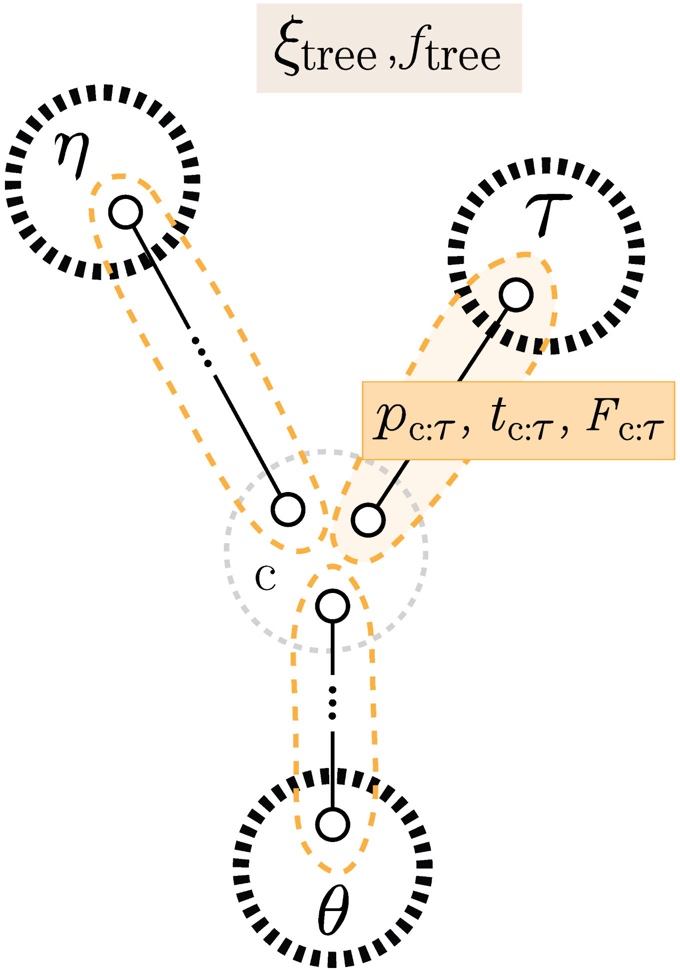




Distributes any graph-state

Meignant, C., Markham, D., & Grosshans, F. (2018) Phys. Rev. A

Multipartite Distribution Metrics



Considering one vector of parameters for each branch connecting the center node to the terminals $\tau \in \mathscr{T}$:

$$\left\{[p_{c:\tau}, t_{c:\tau}, F_{c:\tau}]\right\}_{\tau\in\mathscr{T}} \mapsto [\xi_{tree}, f_{tree}]$$

$$\xi_{tree} = \frac{1}{T_{tree}} \quad , \quad T_{tree} = 2 \cdot \frac{\max_{\tau\in\mathscr{T}}\{t_{c:\tau}\}}{\prod_{\tau\in\mathscr{T}} p_{c:\tau}}$$

$$f_{tree} = \frac{1}{2}\left[\prod_{\tau\in\mathscr{T}} \frac{1+2F_\tau}{3} + \prod_{\tau\in\mathscr{T}} \frac{2(1-F_\tau)}{3} + \prod_{\tau\in\mathscr{T}} \frac{4F_\tau - 1}{3}\right]$$

So far:

✓ Physical description of the network and entanglement distribution

- ✓ Bipartite
- ✓ Multipartite

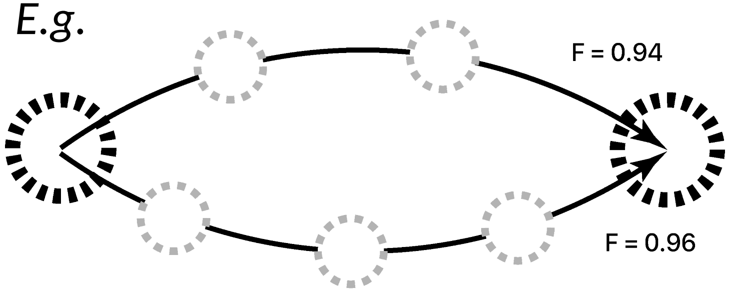
Now:

❖ We need a formalism and algorithms to implement these schemes optimally

Routing

- Routing is solving the problem of finding the shortest path in a given scenario.

E.g.



- Our problem can be formulated as a type of routing that, instead of finding the best path, finds the optimal way to do a protocol optimizing some given metrics.

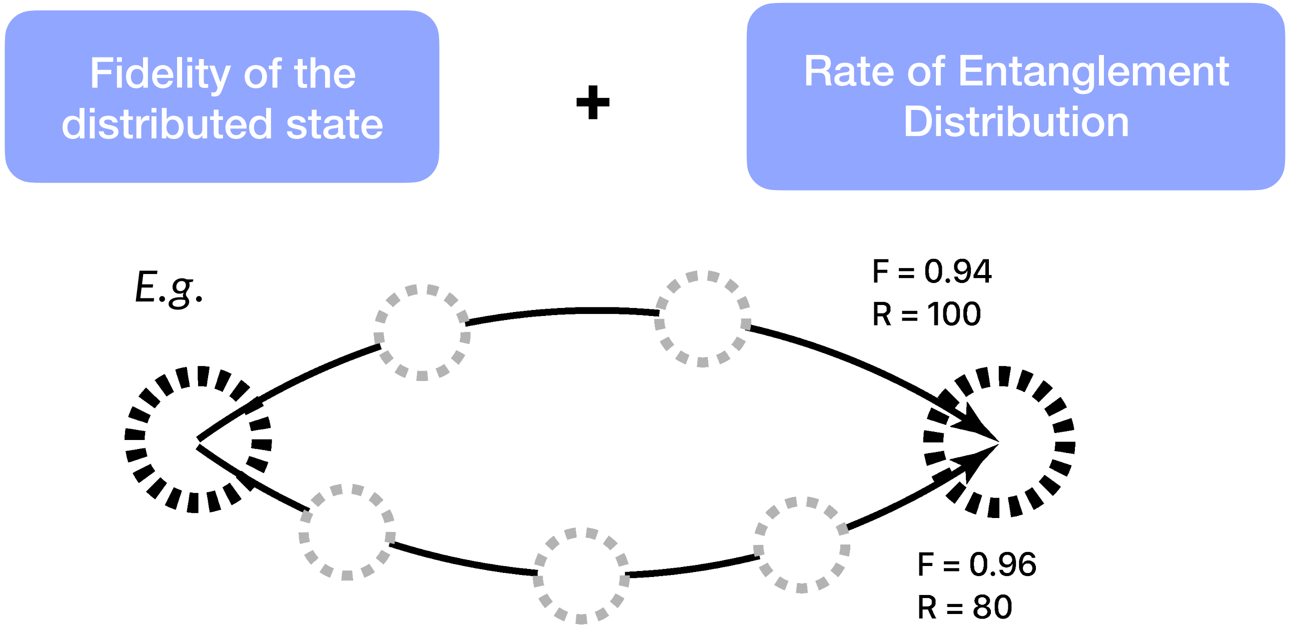
We first introduce the algebra, then the algorithm, and finally prove the optimality of the algorithm using the routing algebra.

Griffin, T. G., & Gurney, A. (2008). Lecture Notes in Computer Science

Sobrinho, J. L. (2005). IEEE/ACM Transactions on Networking

Multi-Objective Routing

To take into account multiple parameters to optimize simultaneously, a multi-objective approach is required. In our work the objectives we are interested in optimizing:



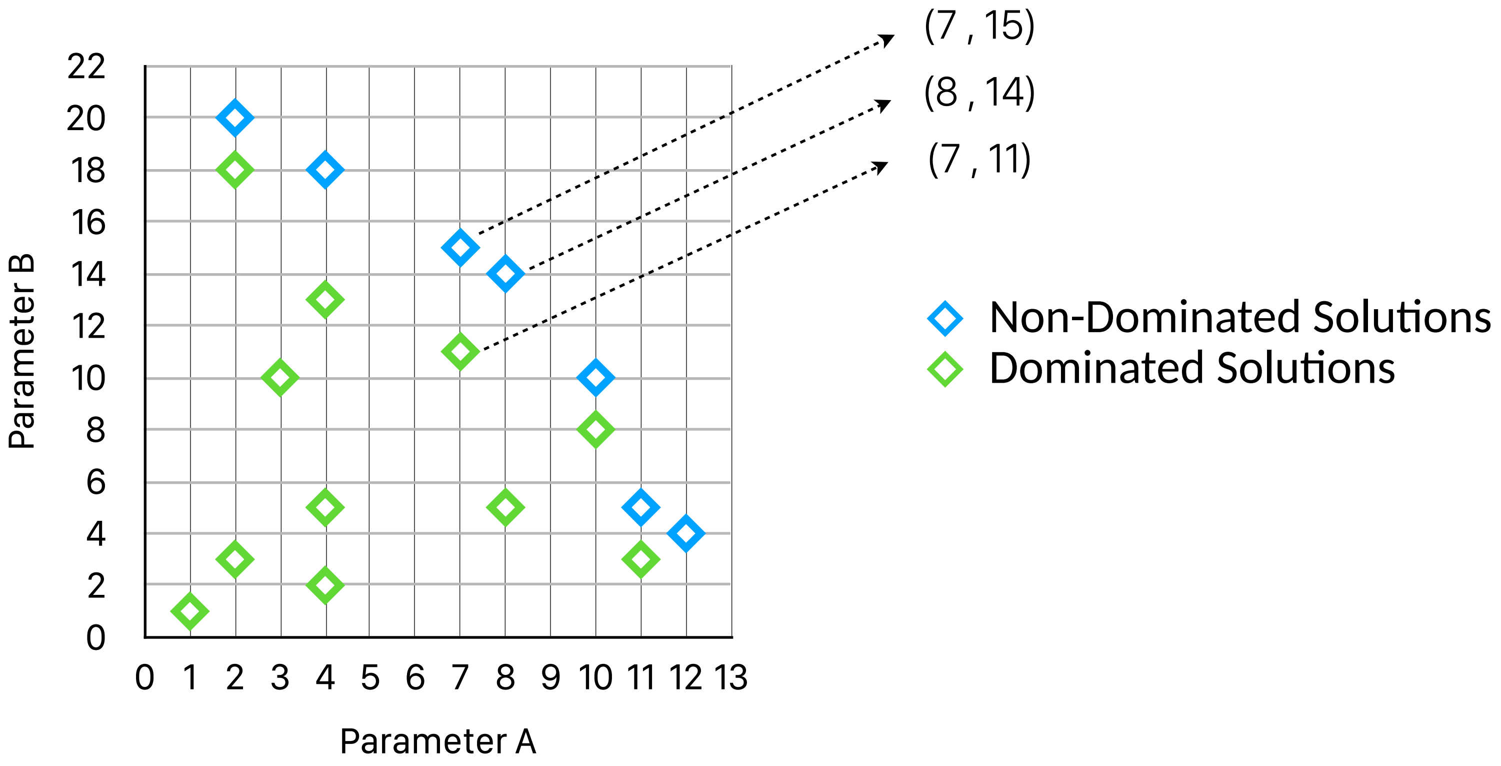
Martins, E. Q. V. (1984). European Journal of Operational Research

Dominance Relation - Pareto Optimality

Definition (informal) *(Dominance)*

let ω and ν be two different paths. ω dominates ν, $\omega\ \mathrm{D}\ \nu$, if ω is better or equal to ν for all the parameters that describe the paths, and the strict order holds at least once.

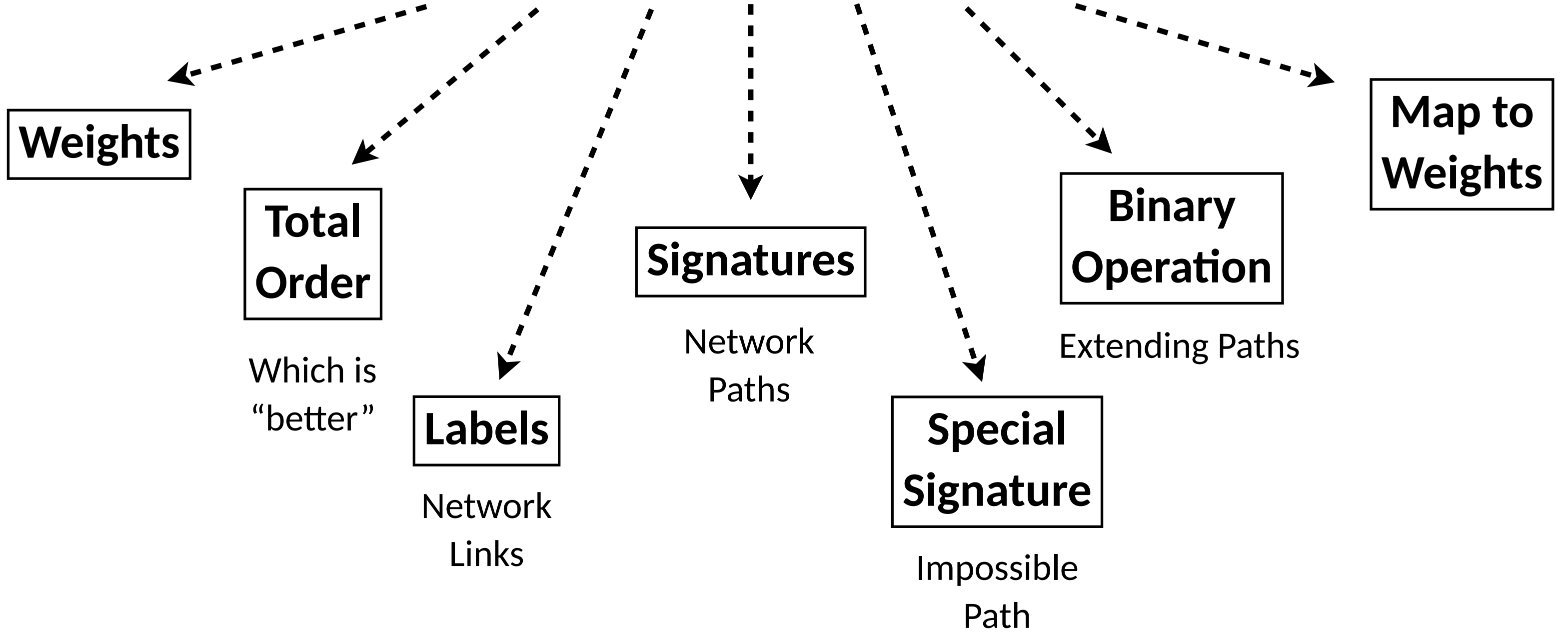
E.g.



Algebras for Routing

routing protocol = routing algebra + algorithm

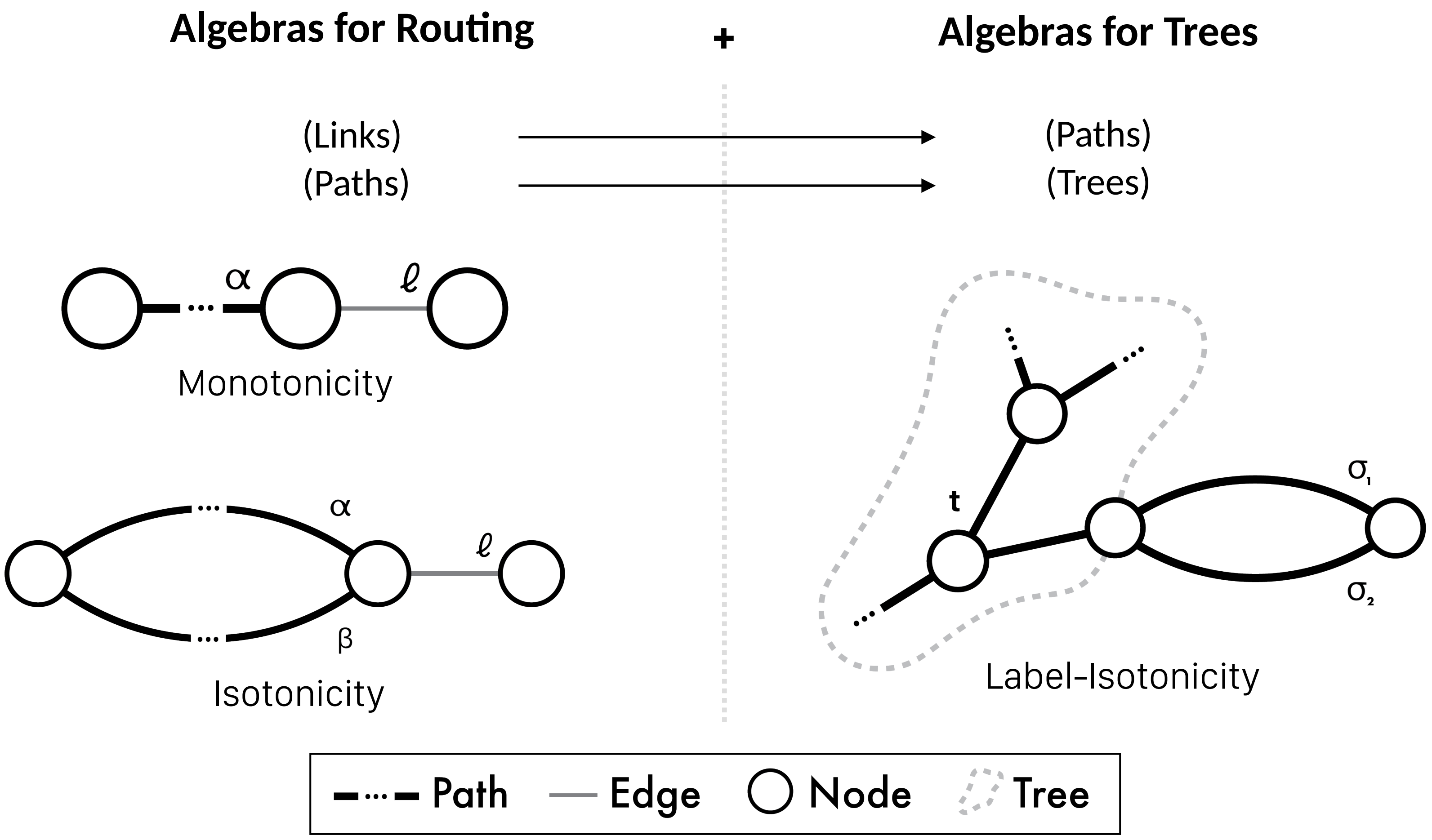
$$(W, \preceq, L, \Sigma, \phi, \oplus, f)$$ **[12]**



Example: Minimum-weight over real numbers $(\mathbb{R} \cap \infty, \leq, \mathbb{R}, \mathbb{R} \cap \infty, \infty, +, \mathrm{id}_\Sigma)$

Sobrinho, J. L. (2005). IEEE/ACM Transactions on Networking

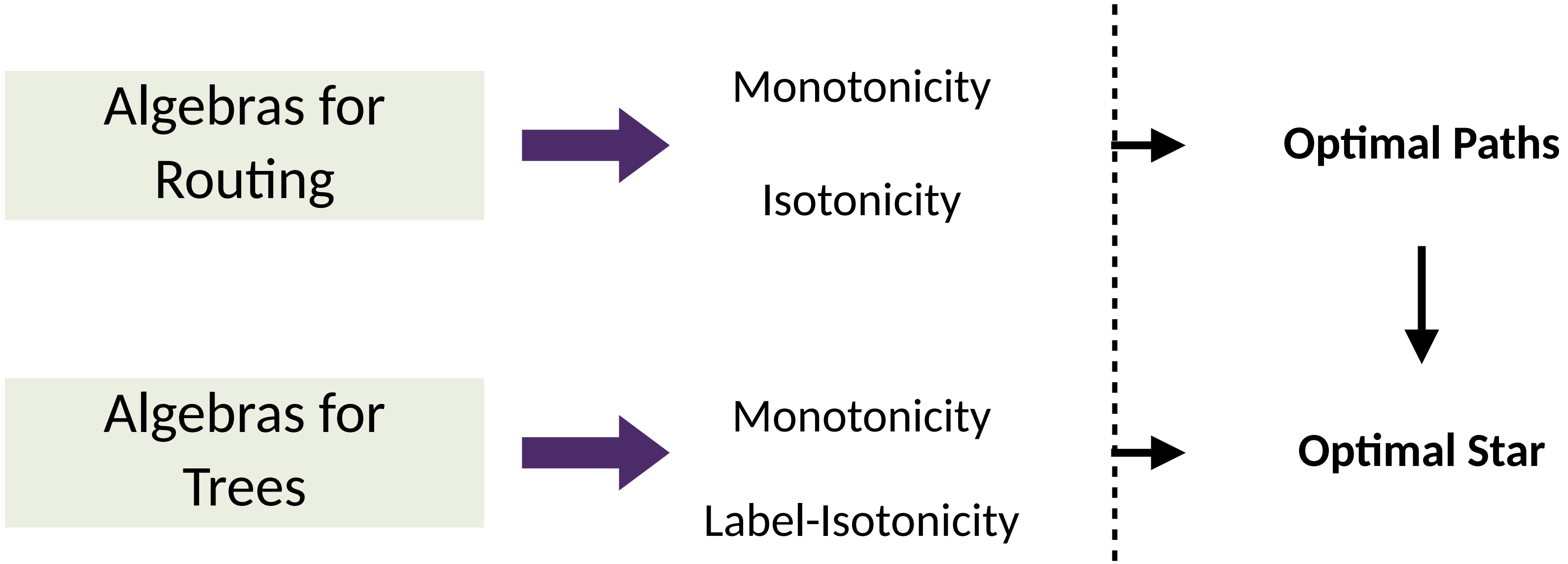
Optimality of the Algorithm



Sobrinho, J. L. (2005). IEEE/ACM Transactions on Networking

Optimality of the Algorithm

Using a multi-objective version of Dijkstra algorithm, and all these properties for the algebras, we can prove that the solution for the 3-GHZ state is optimal:



Simulations over Random Networks

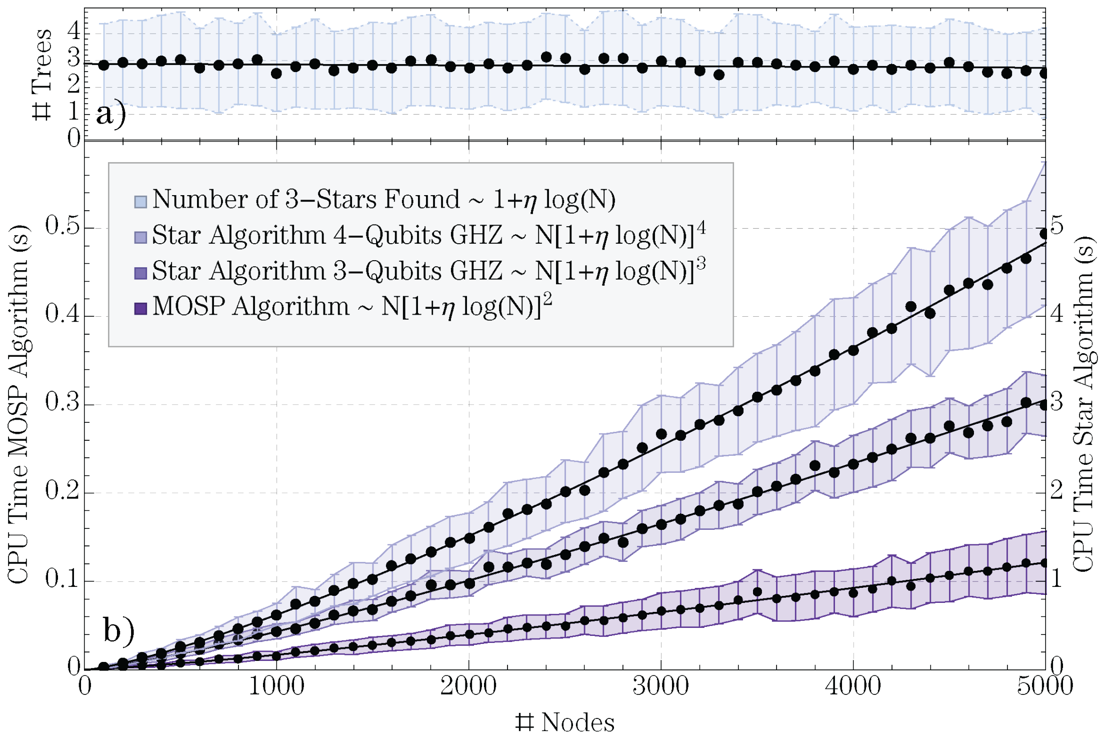


Fig. 1: Algorithm run times for Erdős–Rényi networks

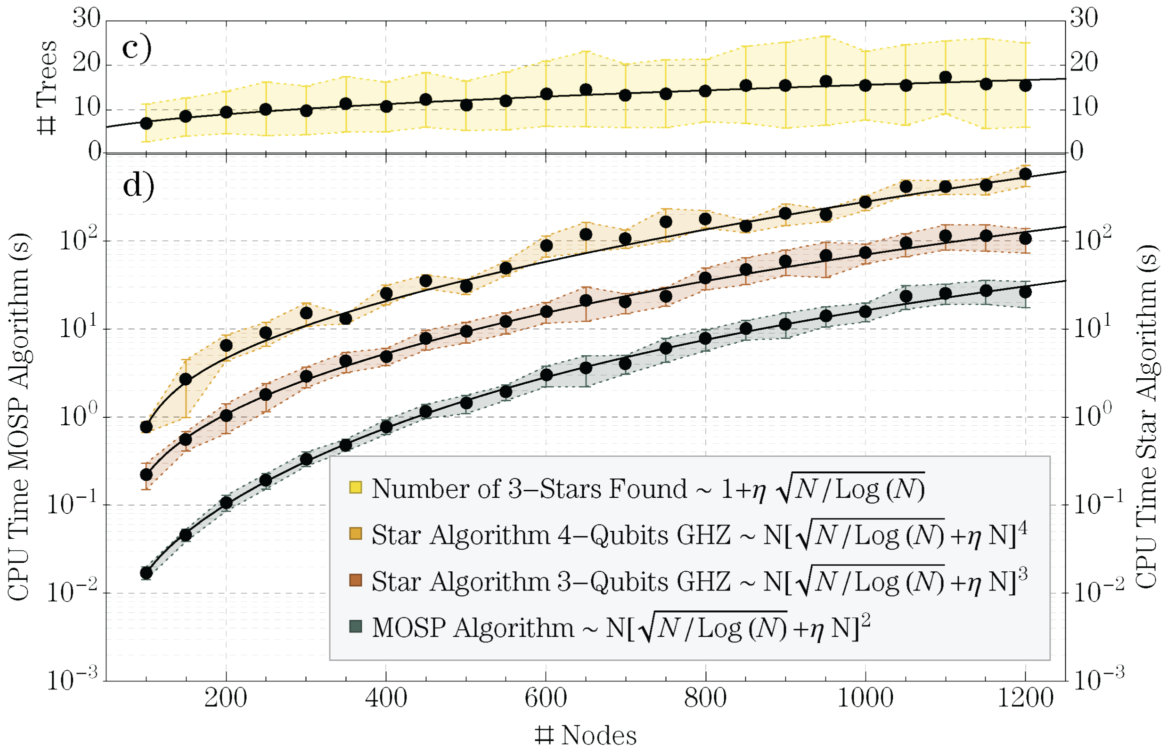


Fig. 2: Algorithm run times for random-geometric networks

L. Bugalho, B. C. Coutinho, F. A. Monteiro, and Y. Omar, Quantum 7,920 (2023)

Simulations over Random Networks

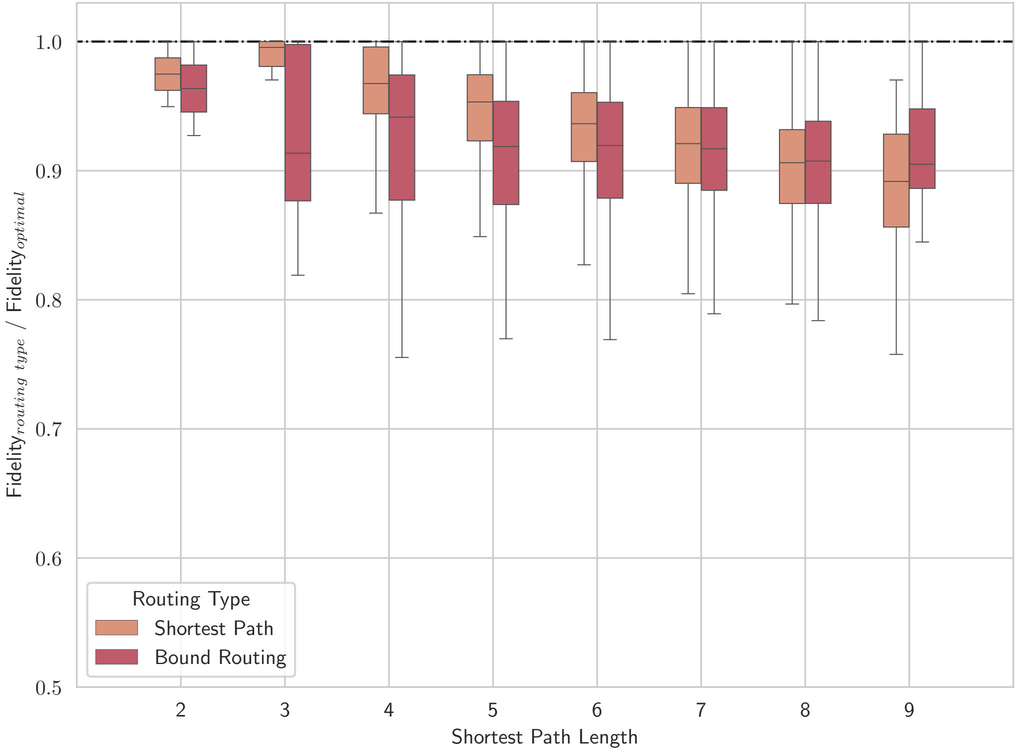


Fig. 3: Comparison between our optimal solution with other methods of routing - fidelity

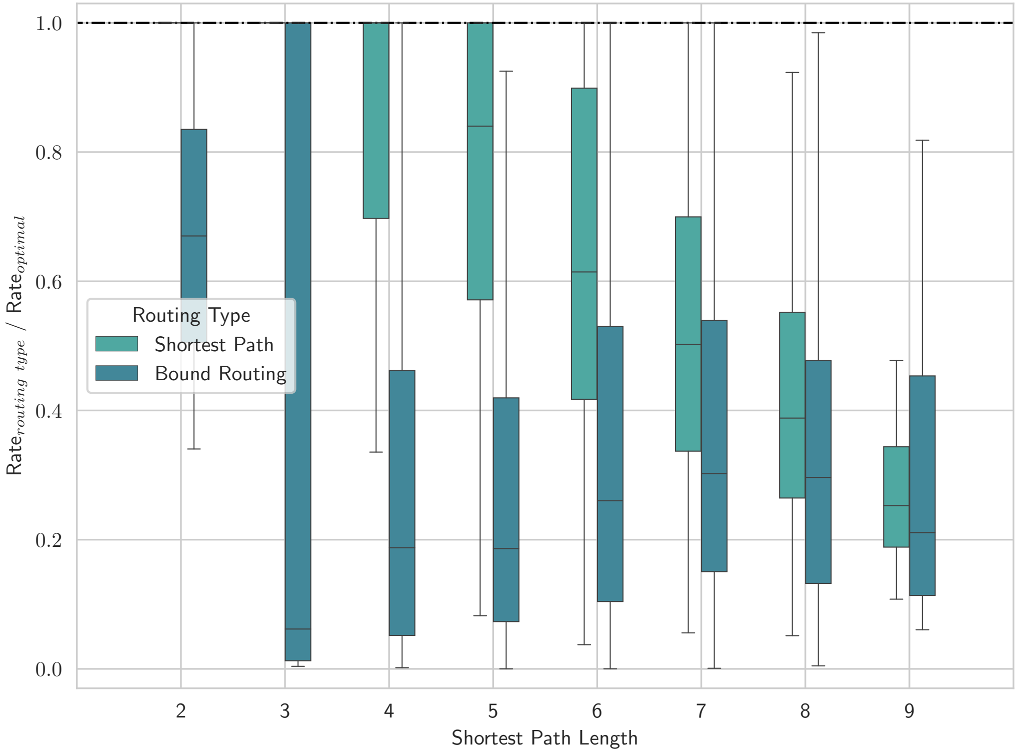


Fig. 4: Comparison between our optimal solution with other methods of routing - rate

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Simulations over Random Networks

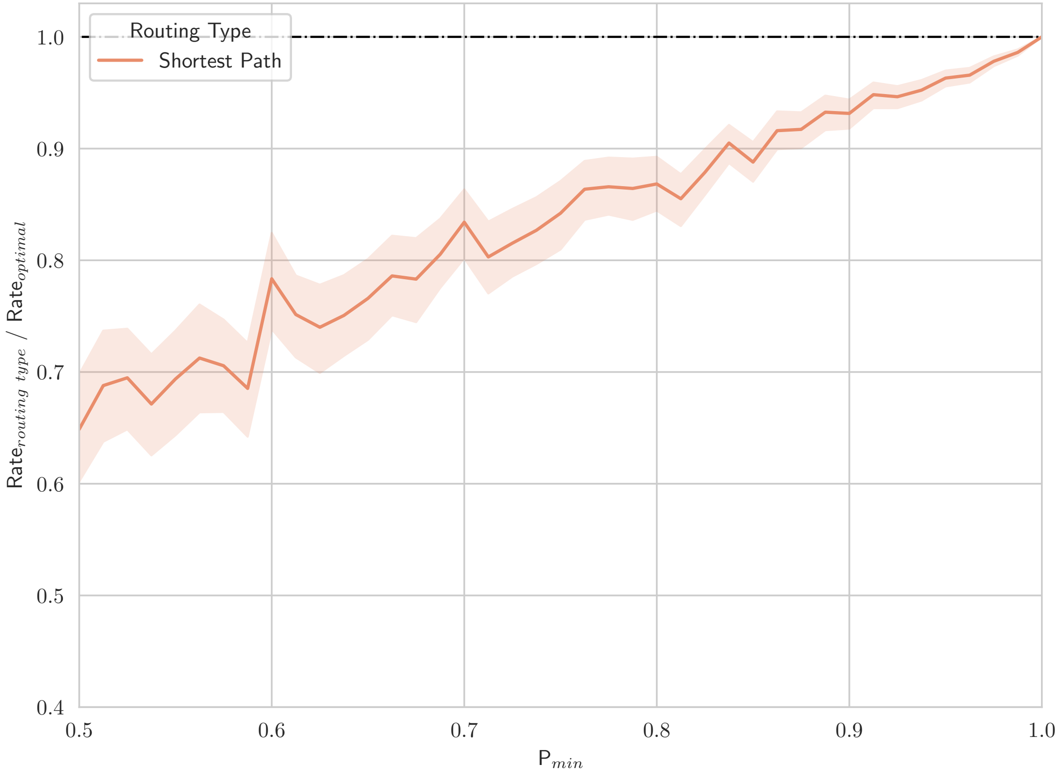

Fig. 5: Convergence between methods of routing, as a function of the distribution of parameters.

L. Bugalho, B. C. Coutinho, F. A. Monteiro, and Y. Omar, Quantum 7,920 (2023)

Distributing Multipartite Entanglement over Noisy Quantum Networks

OUR RESULT:

Find the optimal way to distribute the shortest-star over a quantum network!

Tools: classical routing theory + new quantum metrics
Output: Optimal Algorithm capable of optimizing multiple parameters simultaneously

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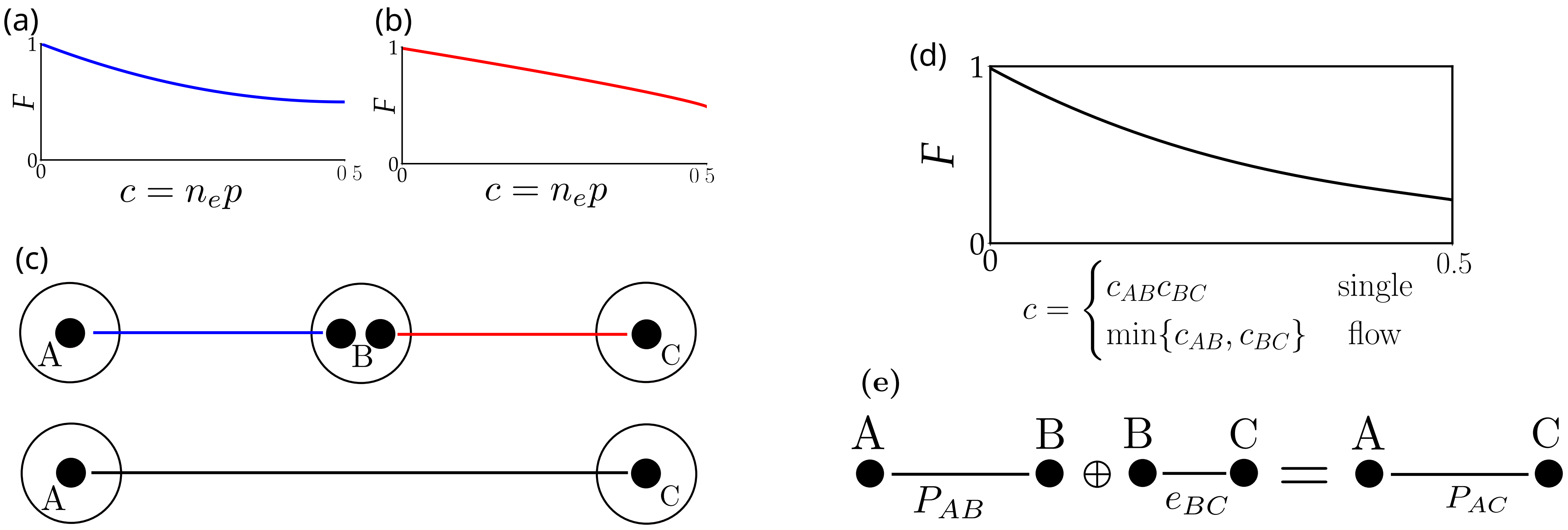
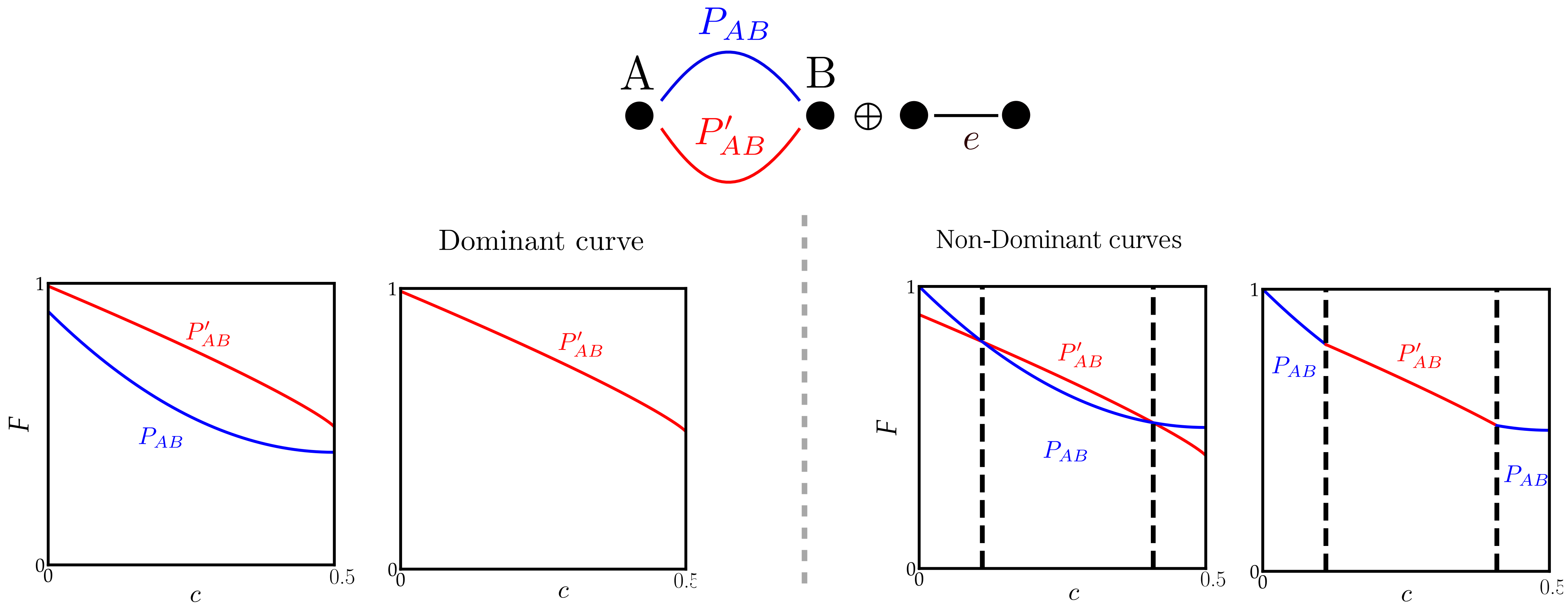
Entanglement Routing Based on Fidelity Curves for Quantum Photonics Channels



B. C. Coutinho, R. Monteiro, L. Bugalho, and F. A. Monteiro, arXiv:2303.12864, (2023)

Entanglement Routing Based on Fidelity Curves for Quantum Photonics Channels



B. C. Coutinho, R. Monteiro, L. Bugalho, and F. A. Monteiro, arXiv:2303.12864, (2023)

Take Home Messages

- Distributing Multipartite Entanglement
  - Optimal algorithms to distribute GHZ and W states of 3 qubits
  - (sub)Optimal polynomial extensions to higher numbers of qubits
  - Maximize multiple parameters at the same time (fidelity, rate, capacity...)
  - Framework capable of generalizations

L. Bugalho, B. C. Coutinho, F. A. Monteiro, and Y. Omar, Quantum 7, 920 (2023)

B. C. Coutinho, R. Monteiro, L. Bugalho, and F. A. Monteiro, arXiv:2303.12864, (2023)

Thank you!

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Acknowledgments

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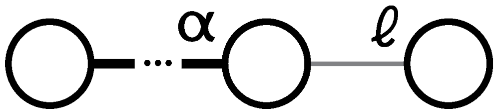
Definition *(Monotonicity)*

an algebra for routing is called monotone if:

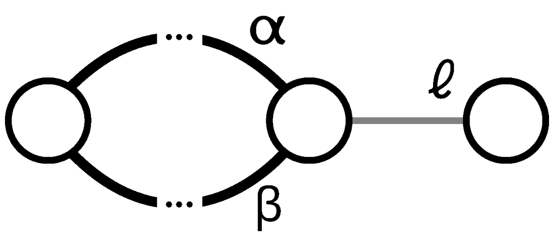
$$\forall\ l \in L, \alpha \in \Sigma : f(\alpha) \preceq f(\alpha \oplus l)$$

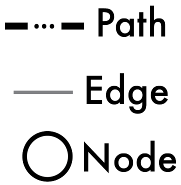


Definition *(Isotonicity)*

an algebra for routing is called isotone if:

$$\forall\ l \in L, \alpha, \beta \in \Sigma : f(\alpha) \preceq f(\beta) \Rightarrow f(\alpha \oplus l) \preceq f(\beta \oplus l)$$

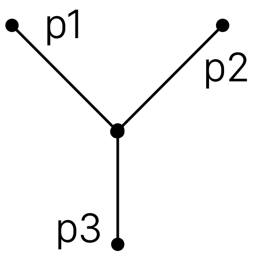




Proposition *(Shortest 3-Tree)*

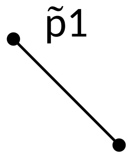
For the shortest-star with 3 terminals, the paths connecting the center node and the terminals must be the shortest-paths, if the underlying algebras for trees are label-isotone.

(Sketch) Proof.

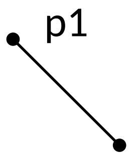


Now suppose p1 is not the shortest-path:

$\exists \tilde{\mathrm{p}}1 :$



$\preceq$



$\equiv$ l.h.s is “better” than r.h.s

Using the definition of label-isotonicity:

$$\forall\ \sigma_1, \sigma_2 \in \Sigma, t \in \Xi : \sigma_1 \preceq \sigma_2 \Rightarrow f(t \oplus \sigma_1) \preceq f(t \oplus \sigma_2)$$

$\implies t \oplus \tilde{\mathrm{p}}1 = \mathrm{p}2 \oplus \mathrm{p}3 \oplus \tilde{\mathrm{p}}1 \preceq t \oplus \mathrm{p}1$, then the best tree contains the shortest path p̃1. Doing the same for every path results in the shortest-tree containing only shortest-paths