Distributing Multipartite Entanglement over Noisy Quantum Networks

Luís Bugalho[1,2], Bruno C. Coutinho[2], Francisco A. Monteiro[2,3] and Yasser Omar[1,4,5]

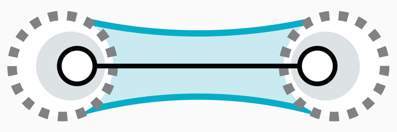
[1]Instituto Superior Técnico, Universidade de Lisboa, Portugal
[2]Instituto de Telecomunicações, Portugal
[3]ISCTE - Instituto Universitário de Lisboa, Portugal
[4]PQI - Portuguese Quantum Institute, Portugal
[5]CeFEMA - Centro de Física e Engenharia de Materiais Avançados, Portugal

arXiv:2103.14759

Quantum Networks

A future **quantum internet** provides support for several quantum tecnhologies, ranging from quantum secure communications to distributed quantum computation and distributed quantum metrology and sensing.
Some applications require the distribution of **multipartite entanglement**, such as quantum conference key agreement (CKA) [1], atomic clocks synchronization [2] and quantum sensor networks [3]. Optimizing its distribution, naturally impacts the applications that utilize this important resource.

Our quantum network model is a graph constituted by a set of nodes and a set of links. Each network link, connecting two neighbouring nodes, contains:



- Noisy quantum channel
- Imperfect quantum memory
- Bipartite entangled state

The protocols for entanglement generation and entanglement swapping are **probabilistic**, and each entangled pair of the network is modelled by a **Werner state**:

$$\rho_W = \gamma|\phi^+\rangle\langle\phi^+| + (1-\gamma)\mathbb{I}$$

where $\gamma = (4F-1)/3$, and F is the fidelity of the state.

Optimising bipartite entanglement distribution over a quantum network is essentially finding the **shortest-path** in a graph. The definition of shortest-path depends on what we want to optimise for.

For bipartite entanglement we ought to maximise the **fidelity of the state** and **probability of success** and minimise the **communications time**.

However, since the fidelity of an entangled pair will depend on the generation fidelity and the decoherence, we must first optimise four parameters individually and then map them to only three:

$$[p_{m:n}, t_{m:n}, \gamma_{m:n}, \sigma_{m:n}] \mapsto [p_{m:n}, t_{m:n}, F_{m:n}]$$

$$\frac{4F_{m:n}-1}{3} = \gamma_{m:n} e^{-t_{m:n}/\sigma_{m:n}}$$

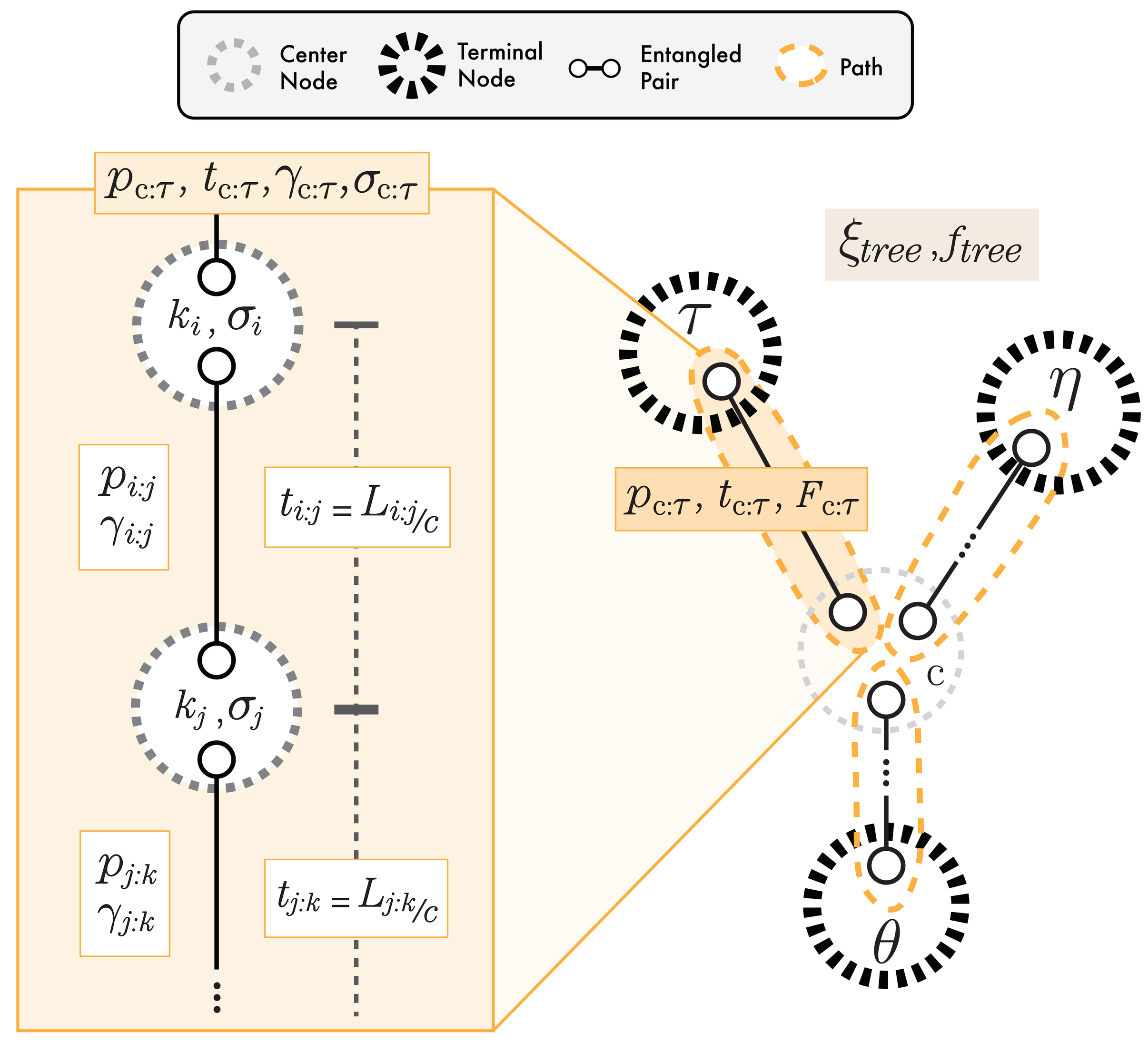


Figure 1. Branch composed by an arbitrary number of links. Each link has a fidelity $\gamma_{i:j}$, a communications time $t_{i:j}$ and probability of successful generation $p_{i:j}$. Moreover, each node has a memory coherence time σ_i and a probability of successful entanglement swapping k_i.

Figure 2. Star composed by 3 different paths, connecting terminal nodes τ, η and θ to the center node c. Notice that the rate ξ_{tree} depends on each branch probability of success $p_{c:\tau}$ and communications time $t_{c:\tau}$. Moreover, the fidelity of the final multipartite state f_{tree} depends on each branch fidelity $F_{c:\tau}$.

Figure 4. Simulations of the MOSP algorithm and the Star Algorithm for 3 and 4 qubits GHZ state distribution in Erdös-Rényi networks with average degree 3. Complexity scaling with the number of network nodes.

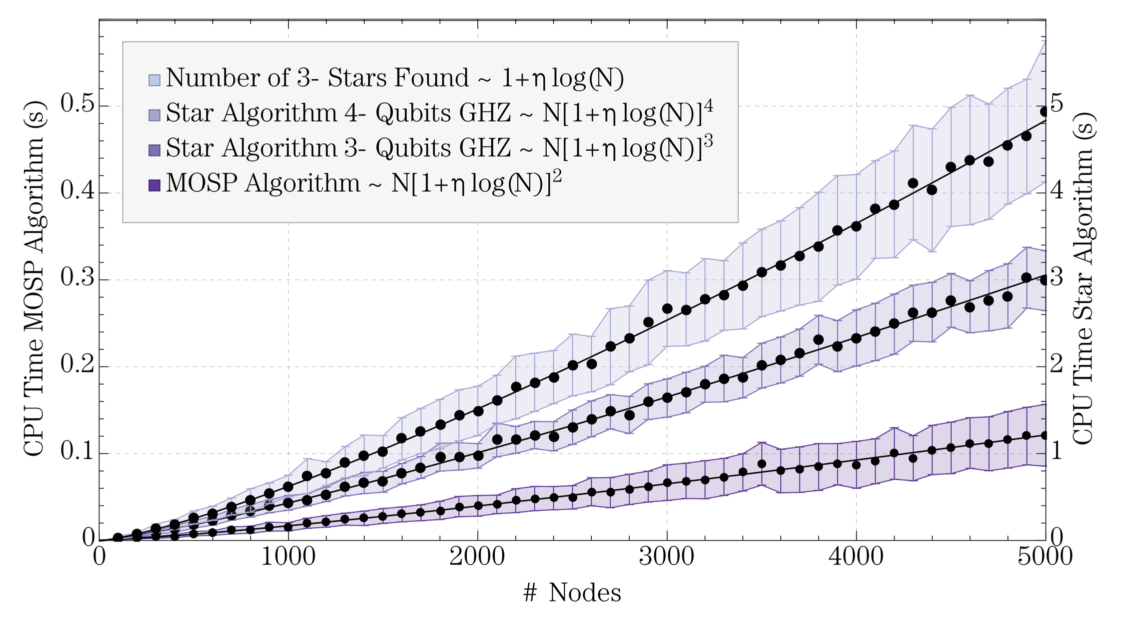
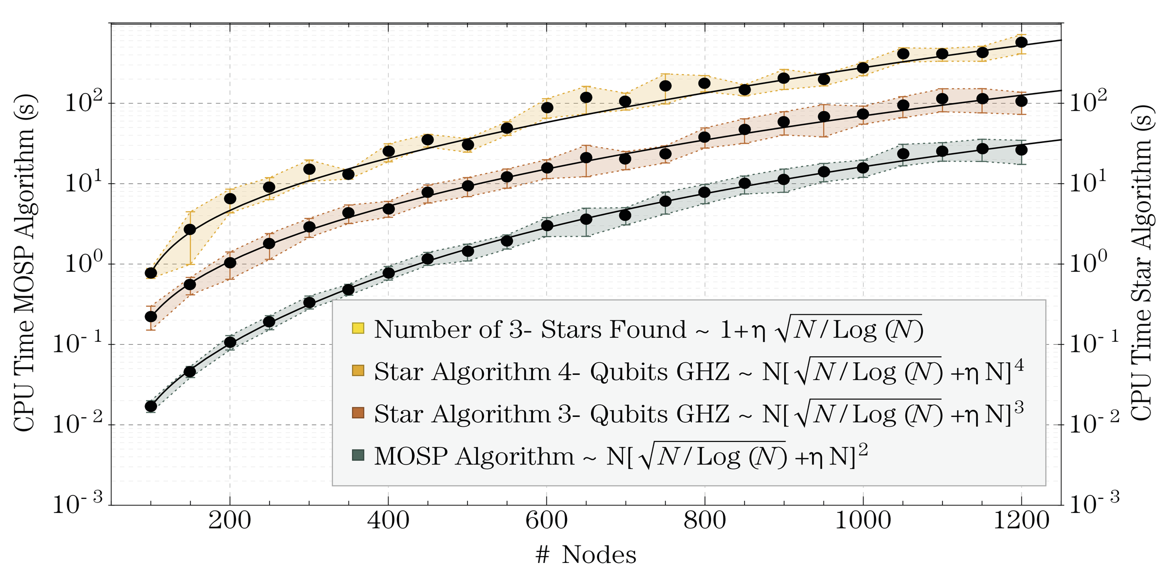


Figure 5. Simulations of the MOSP algorithm and the Star Algorithm for 3 and 4 qubits GHZ state distribution in random geometric networks with average degree 8. Complexity scaling with the number of network nodes.



Multipartite Entanglement Distribution

Similarly to [4], to distribute a 3-qubit GHZ state, first we have to find the **shortest-tree** connecting the set of terminal nodes, and then perform a set of measurements on some qubits. For 3 qubits, the shortest-tree is a **star-graph**, and the scheme is equivalent to first distributing bipartite entanglement between every terminal and a center node and then performing a set of operations on the center node.

Thus, the central idea is **finding the center node** for which this results in the optimal solution. Optimality is defined concerning the **metrics for the distribution** of the final multipartite entangled state, namely the **rate of distribution** and the **fidelity**.

The average time for distribution depends on the maximum of each path's communications time multiplied by the average number of attempts before the first success. The **average rate** would then be the inverse of:

$$T_{tree} = 2 \cdot \frac{\max_{\tau \in \mathcal{T}}\{t_{c:\tau}\}}{\prod_{\tau \in \mathcal{T}} p_{c:\tau}} \quad , \quad \xi_{tree} = \frac{1}{T_{tree}}$$

The form of the final distributed state is equivalent to applying a depolarising channel to each qubit of the GHZ state, correspondent to each branch connecting the center node to a terminal. The **final fidelity of this state** is described by:

$$f_{tree} = \frac{1}{2}\left[\prod_{\tau \in \mathcal{T}} \frac{1+2F_\tau}{3} + \prod_{\tau \in \mathcal{T}} \frac{2(1-F_\tau)}{3} + \prod_{\tau \in \mathcal{T}} \frac{4F_\tau - 1}{3}\right]$$

Multi-objective Routing Algorithm

In our problem, we want to optimise two distinct parameters. For that, we require a multi-objective approach. In 1984, Martins [5] introduced **multi-objective shortest-paths** (MOSP) algorithms, defining the dominance relation and using the **Pareto optimality** definition – optimal paths are non-dominated paths. This results in a set of optimal solutions that is usually larger than one – the **Pareto front**.

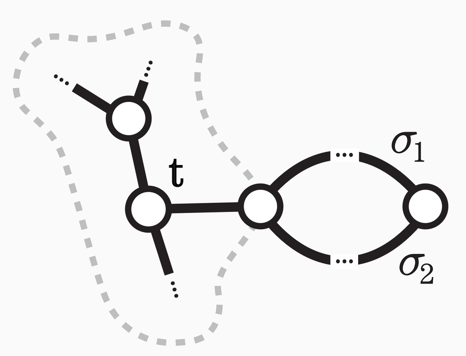
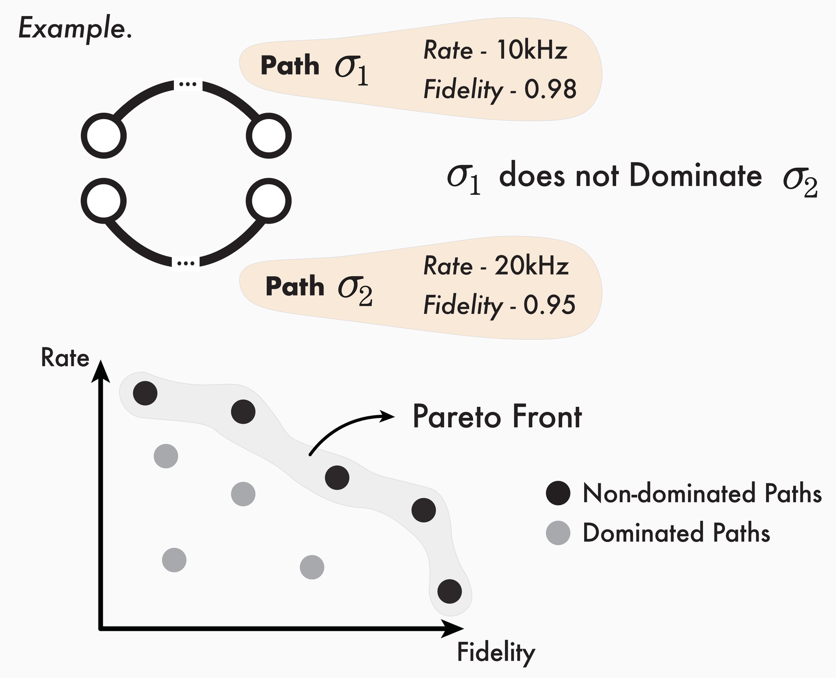


Figure 3. Label-Isotonicity: for every tree and path, when extending a tree with a path, choosing a better path results in a better tree.

Example.



In [6] Sobrinho introduces a more fundamental definition for these metrics involved in routing problems – **algebras for routing**. These algebras incorporate the parameters of the links, paths and how paths are extended with links – **the metrics** –, allowing for a bigger level of abstraction.

Multi-objective routing – **one algebra for each parameter** (rate, fidelity,...). To prove optimality, we expanded these algebras for the case of trees (instead of paths) and introduced **label-isotonicity** *(Figure 3)*.

Results and Conclusions

> We were able to calculate the effect of **imperfect entanglement** throughout quantum networks on the distribution of GHZ and W states, importante resources for distributed applications, such as **communication** and **sensing**;

> We defined a metric for the fidelity and realise an algorithm with **polinomial runtime** for distributing these states optimally, **maximising simultaneously the fidelity and the rate of distribution**;

> Using classical techniques and tools, we can **prove the optimality** and create the necessary formalism to expand the methodology, while also providing the **necessary conditions to attain this optimality**.

References:

[1] G. Murta, F. Grasselli, H. Kampermann and D. Bruß, Adv. Quantum Technol., vol. 3, p. 2000025 (2020)
[2] P. Kómár et al., Nat. Phys., vol. 10, no. 8, pp. 582–587, (2014)
[3] Z. Eldredge, M. Foss-Feig, J. A. Gross, S. L. Rolston and A. V. Gorshkov, Phys. Rev. A, vol. 97, no. 4, p. 042337 (2018)
[4] C. Meignant, D. Markham, and F. Grosshans, Phys. Rev. A, vol. 100, no. 5, p. 052333 (2019)
[5] E. Q. V. Martins, Eur. J. Oper. Res., vol. 16, no. 2, pp. 236–245 (1984)
[6] J. L. Sobrinho, Comput. Commun. Rev., vol. 33, no. 4, pp. 49–60 (2003)

For further details:

[7] Luís Bugalho, Bruno C. Coutinho, Francisco A. Monteiro and Yasser Omar, arXiv:2103.14759 (2021)

Acknowledgments:

The authors thank the support from Fundação para a Ciência e a Tecnologia (Portugal), namely through project UIDB/50008/2020 and QuantSat-PT, as well as from projects TheBlinQC and QuantHEP supported by the EU H2020 QuantERA ERA-NET Cofund in Quantum Technologies and by FCT (QuantER-A/0001/2017 and QuantERA/0001/2019, respectively), and from the EU H2020 Quantum Flagship projects QIA (820445) and QMiCS (820505) and to project UIDB/04540/2020.

About the Authors:


Luís Bugalho
email: luis.bugalho@tecnico.ulisboa.pt


Bruno C. Coutinho
email: bruno.coutinho@lx.it.pt


Francisco A. Monteiro
email: francisco.monteiro@lx.it.pt


Yasser Omar
email: yasser.omar@tecnico.ulisboa.pt